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Scaling of the quantum Hall plateau-plateau transition in graphene

A. J. M. Giesbers,[1] U. Zeitler,[1,*] L. A. Ponomarenko,[2] R. Yang,[2] K. S. Novoselov,[2] A. K. Geim,[2] and J. C. Maan[1]
[1]*High Field Magnet Laboratory, Institute for Molecules and Materials, Radboud University Nijmegen, Toernooiveld 7, 6525 ED Nijmegen, The Netherlands*
[2]*Department of Physics and Astronomy, University of Manchester, M13 9PL Manchester, United Kingdom*
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We show that the width of the longitudinal magnetoconductivity peaks in graphene related to the $N=1$ Landau level displays a power-law type temperature dependence, $\Delta\nu \propto T^{\kappa}$, with $\kappa=0.37\pm 0.05$. Similarly, the derivative of the Hall conductivity at the plateau transition, $(d\sigma_{xy}/d\nu)$, scales as $T^{-\kappa}$ with $\kappa=0.41\pm 0.04$ for both the first and second Landau levels of electrons and holes. These results confirm the universality of a critical quantum Hall scaling in the higher Landau levels of graphene. In the zeroth Landau level, however, $\Delta\nu$ and $d\sigma_{xy}/d\nu$ are essentially temperature independent, pointing toward a different type of scaling that is possibly governed by a temperature independent intrinsic length.

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The integer quantum Hall effect in two-dimensional electron systems (2DESs) is caused by localized states in the tails of individual Landau levels which give rise to quantized plateaus in the Hall resistance. The states in the center of the Landau levels are extended; their wave functions are delocalized. Their delocalization is governed by a localization length, which decays exponentially away from the Landau level centers[1,2] with a universal critical scaling exponent related to this decay.[3]

In this Rapid Communication we investigate the scaling behavior of the quantum Hall plateau-plateau transitions in the recently discovered new type of 2DES, graphene.[4,5] When changing the carrier concentration n at a constant field, the peak width of the longitudinal conductivity for higher Landau levels ($|N|\geq 1$) and the inverse slope of the Hall conductivity scale as T^{κ}. Our experimentally measured scaling exponent $\kappa=0.40\pm 0.02$ is consistent with universal scaling theory.[1–3,6–8] The transition through the zeroth Landau level, however, shows no clear scaling behavior which we explain by a different scaling mechanism governed by a temperature independent intrinsic length scale.

Our sample was made by micromechanical exfoliation of natural graphite and subsequently contacted by gold contacts and patterned into a 1-μm-wide Hall bar by electron-beam lithography and reactive plasma etching.[9] The structure was deposited on a 300 nm Si/SiO_2 substrate thereby forming a graphene ambipolar field effect transistor. Prior to the measurements the sample was annealed at 400 K, placing its charge neutrality point (CNP) at zero gate voltage with a mobility of $\mu=1.0\ \text{m}^2\ (\text{V s})^{-1}$.

In a magnetic field the density of states (DOS) in graphene splits up into nonequidistant Landau levels,[4,5,10,11]

$$E_N = \pm\sqrt{2e\hbar v_F^2 B|N|} \tag{1}$$

(see inset of Fig. 1). $N=0,\pm 1,\pm 2,\ldots$ identifies the fourfold degenerate Landau levels of electrons and holes. This Landau level spectrum leads to the half-integer quantum plateaus $\sigma_{xy}=4(N+1/2)ie^2/h$ accompanied by zero minima in the longitudinal conductivity σ_{xx} (Fig. 1). Around the centers of the Landau levels the states are extended and the Hall conductivity changes to its next plateau, while the longitudinal conductivity displays a peak.[12]

The region of delocalized states in the center of a Landau level can be described by the energy interval where the localization length $\xi(E)$, i.e., the spatial extension of the wave function, increases beyond some characteristic length, $\xi(E)>L$, and states remain localized when $\xi(E)$ remains below L. According to scaling theory the localization length $\xi(E)$ follows a power-law behavior[1,2] as a function of energy,

$$\xi(E) = \xi_0|E-E_c|^{-\gamma}. \tag{2}$$

E_c is the energy of a Landau level center and $\gamma\approx 2.3$ is the critical exponent.

To probe this delocalization we have to translate the abstract dependence of $\xi(E)$ on energy to measurable quantities. In a first approximation, we assume a constant DOS

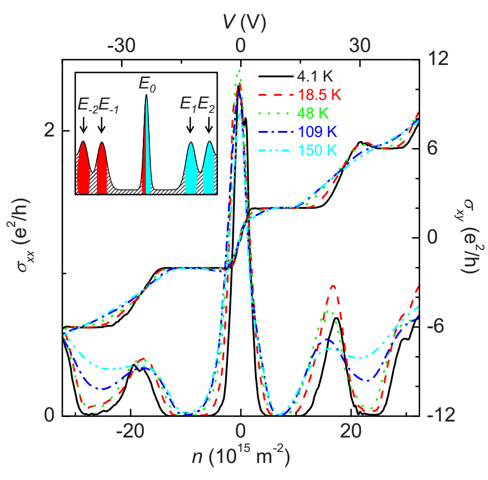

FIG. 1. (Color online) Longitudinal conductivity σ_{xx} and Hall conductivity σ_{xy} as a function of concentration n (bottom axis) and voltage V (top axis) at $B=20$ T for different temperatures. The conductivities were calculated by tensor inversion from the measured symmetrized resistivities for both magnetic field orientations. The inset shows the Landau level spectrum in graphene with the dashed regions indicating the localized states between two Landau levels.

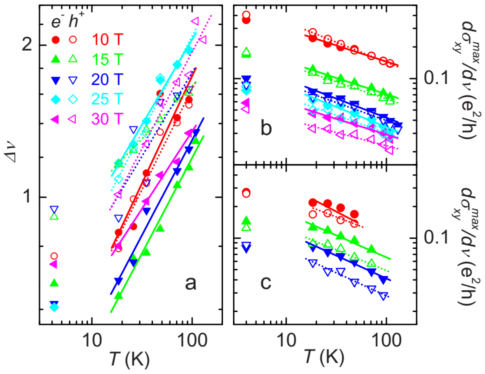

FIG. 2. (Color online) (a) $\Delta\nu$, distance between two extrema in the derivative $d\sigma_{xx}/d\nu$ of the first electron Landau level (solid symbols) and the first hole (open symbols) Landau level for different magnetic fields. (b) The derivative $(d\sigma_{xy}/d\nu)^{\max}$ as a function of temperature for the same levels. (c) The derivative $(d\sigma_{xy}/d\nu)^{\max}$ as a function of temperature for the second electron and hole level.

close to E_c.[13] In this case the energy in Eq. (2) is directly proportional to the measurable filling factor, $\nu = nh/eB$.

At finite temperatures the characteristic length L is determined by the inelastic scattering rate, $L_{in} \propto T^{-p/2}$, with p as the inelastic scattering exponent. We can directly relate this temperature dependent quantity to the scaling of the localization length, and Eq. (2) yields $|\nu - \nu_c| \propto T^{p/2\gamma}$, with ν_c as the Landau level center and ν as its localization edge. This scaling behavior has therefore a direct relation to the quantum Hall transitions measured in the magnetoconductivity at finite temperatures through the quantities $(d\sigma_{xy}/d\nu)^{\max}$ and $\Delta\nu$.[1,2] The width $\Delta\nu$ is defined as the distance between two extrema in the derivative $d\sigma_{xx}/d\nu$, one on each side of a conductivity peak of a particular Landau level in Fig. 1, and $(d\sigma_{xy}/d\nu)^{\max}$ is the maximum in the derivative of σ_{xy}. Both quantities show a power-law behavior $\Delta\nu \propto T^{\kappa}$ and $(d\sigma_{xy}/d\nu)^{\max} \propto T^{-\kappa}$ with the critical exponents κ and γ related according to $\kappa = p/2\gamma$. Though an exact value for p cannot be determined from our data, $p=2$ is commonly used for two-dimensional systems governed by short range scattering,[6,14,15] which is indeed expected in our type of graphene sample.[11,16–18]

Since the higher Landau levels in graphene have been shown to behave similarly to traditional 2DESs,[19] they are a good starting point for scaling measurements. The width $\Delta\nu$ and derivative $(d\sigma_{xy}/d\nu)^{\max}$ as a function of temperature for the first and second electron and hole Landau levels are shown in Fig. 2 at fixed magnetic fields between 5 and 30 T. The temperature dependence of the transition widths $\Delta\nu$ for the first electron and hole levels at magnetic fields between 10 and 30 T indeed display a power-law behavior following $\Delta\nu \propto T^{\kappa}$. The scaling exponents extracted from these data, $\kappa = 0.37 \pm 0.05$ for holes and $\kappa = 0.37 \pm 0.06$ for electrons, are all identical within the error margins and show no evidence of a magnetic field dependence. The error is determined by the scattering of all the individual κ values; the statistical error for each κ is smaller. At $T < 15$ K the curves in Fig. 2(a) flatten, which is an indication that the localization length becomes independent of temperature and becomes dominated by an intrinsic length scale,[6] possibly the 1 μm width of the sample.

Also the scaling behavior of $(d\sigma_{xy}/d\nu)^{\max}$ between the $\nu = \pm 2$ and $\nu = \pm 6$ plateaus is consistent with these observations, as shown in Fig. 2(b). For both the $N=-1$ hole level and the $N=1$ electron level the curves show a power-law behavior with scaling exponents $\kappa = 0.40 \pm 0.03$ and $\kappa = 0.40 \pm 0.04$, respectively, for all fields up to $B = 25$ T.

In the high field limit ($B = 30$ T) the data indicate a small reduction of the scaling exponent. At these high fields the degeneracy of the Landau levels is partly lifted,[20] which leads to an additional minimum in σ_{xx} and a developing plateau in σ_{xy} in the center of the Landau levels ($\nu = \pm 4$). Due to the broad Landau levels this splitting remains obscured in σ_{xx} and σ_{xy}. However, their derivatives at the center Landau level positions are much more sensitive to the onset of this splitting, which makes it difficult to extract reliable scaling data at the highest magnetic field.

All curves for $(d\sigma_{xy}/d\nu)^{\max}(T)$ start to flatten off at low temperatures, similar to what is observed in the width [see Fig. 2(b)]. This confirms that an intrinsic length apparently starts to dominate the localization length below $T = 15$ K.

Figure 2(c) shows $(d\sigma_{xy}/d\nu)^{\max}$ for the second hole and electron Landau levels. Despite of the limited temperature range it does show a power-law behavior with scaling exponents of $\kappa = 0.40 \pm 0.03$ and $\kappa = 0.41 \pm 0.03$ for holes and electrons, respectively. The width at the second Landau level is no longer clearly distinguishable and therefore no data could be obtained on the scaling behavior of these levels from this technique.

When we assume the temperature exponent of the inelastic scattering length to be $p=2$, we obtain by the relation $\kappa = p/2\gamma$ a critical exponent $\gamma = 2.5 \pm 0.2$ for all the higher Landau levels combined. This value is in good agreement with the universal value $\gamma = 2.39 \pm 0.1$ showing that localization of the higher Landau levels in graphene follows a similar scaling behavior as traditional two-dimensional electron systems.

These results are also consistent with the low temperature behavior of the conduction in the Landau level tails.[13] In these tails the conductivity decreases with decreasing temperature and disappears when the temperature is lowered to $T=0$ (see Fig. 1). When kT is small enough to make the activation to the mobility edge and excitation across potential barriers to neighboring states improbable, conduction is governed by a variable range hopping type of conductivity. In this regime electrons or holes are able to tunnel between states within an energy range kT leading to a slightly increased conductivity. The temperature dependence of the conductivity in this regime is given by[21–25]

$$\sigma_{xx} = \sigma_0 e^{-\sqrt{T_0/T}}, \tag{3}$$

with a temperature dependent prefactor $\sigma_0 \propto 1/T$. The characteristic temperature T_0 is determined by the Coulomb energy and is inversely proportional to the localization length $\xi(\nu)$ at a particular filling factor ν,

$$T_0(\nu) = C \frac{e^2}{4\pi\epsilon\epsilon_0 k_B \xi(\nu)}, \tag{4}$$

with C as a dimensionless constant in the order of unity and $\epsilon \approx 2.5$ is the effective dielectric constant for graphene on silicon dioxide.

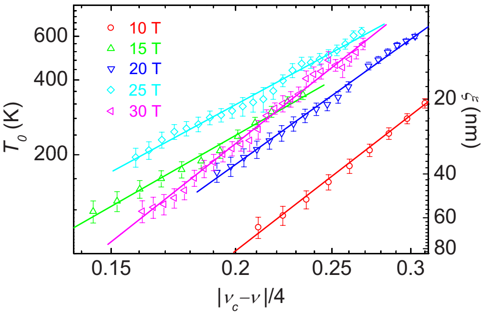

FIG. 3. (Color online) Scaling behavior in the tails of the first electron Landau level for various magnetic fields as a function of relative filling factor $|\nu_c-\nu|/4$ with $\nu = nh/eB$. The factor 1/4 in the x axis accounts for the fourfold Landau level degeneracy. The localization length on the right axis was calculated by Eq. (4) with $C=1$.

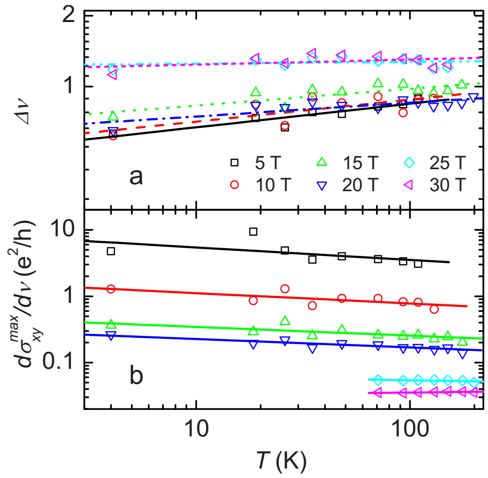

FIG. 4. (Color online) (a) $\Delta\nu$, distance between two extrema in the derivative $d\sigma_{xx}/d\nu$ as a function of temperature for the zeroth Landau level. (b) The derivative $(d\sigma_{xy}/d\nu)^{\max}$ as a function of temperature for the zeroth Landau level.

Using Eq. (3) we can extract a value for the characteristic temperature T_0 from the temperature dependent conductivity measurements, which is directly related to the localization length via Eq. (4). Figure 3 shows the results for the first electron Landau level as a function of the relative filling factor $|\nu_c-\nu|$. A similar graph is found for the first hole level. Combining these results with Eq. (2) yields a critical exponent $\gamma=2.6\pm0.5$ for the $N=\pm1$ Landau level. This is indeed in good agreement with the values obtained from the width of the $\sigma_{xx}(\nu)$ peak and the maximum slope of $\sigma_{xy}(\nu)$.

It is interesting to note that the universality of the critical exponent is preserved despite the fourfold degeneracy of the Landau levels in graphene compared to lower degeneracies measured in other 2DESs. It was argued that degenerate levels would show a change in the critical exponent.[26] The results presented here, however, show that the Landau level degeneracy does not appear to be relevant.[27]

Let us now focus on the $N=0$ Landau level. From a similar analysis as described before we have extracted $\Delta\nu$ and $(d\sigma_{xy}/d\nu)^{\max}$ as a function of temperature for magnetic fields between 5 and 30 T; the results are shown in Fig. 4. Contrary to the higher Landau levels, neither $\Delta\nu$ width [Fig. 4(a)] nor $(d\sigma_{xy}/d\nu)^{\max}$ [Fig. 4(b)] depends on temperature.

Note that in the low temperature limit at high magnetic fields scaling measurements of the zeroth Landau level, especially in $(d\sigma_{xy}/d\nu)^{\max}$, are obscured, similar to what we observed for the $\nu=\pm4$ state in the first Landau level. In this regime the Landau level degeneracy is partly lifted and a $\nu=0$ state appears[28] and develops into a quantum Hall plateau at the CNP. Consequently $(d\sigma_{xy}/d\nu)^{\max}$ will go to zero and is unrelated to any scaling behavior.

The temperature independence of $\Delta\nu$ and $(d\sigma_{xy}/d\nu)^{\max}$ in the zeroth Landau level suggests that the localization mechanism in the zeroth Landau level significantly differs from traditional scaling theories. In particular, localization does not seem to be cut off by a thermal length but remains intact up to room temperature.[29] Within the framework of standard scaling theories we might describe our observation by a temperature independent intrinsic length L_{int} governing localization (rather than the thermal length). L_{int} is directly related to the localization length by $L_{int}\approx\xi=\xi_0|\Delta\nu/2|^{-\gamma}$, where the prefactor ξ_0 is proportional to the magnetic length $l_B=\sqrt{\hbar/eB}$ with a proportionality factor of the order of 1.[3] This provides an estimate for the intrinsic length at high magnetic fields ($B>20$ T) of the order of 10 nm. At lower fields where localization becomes poorer the characteristic length extracted form our data increases to about 90 nm at 5 T. At these fields also the curves in Fig. 4 start to display a slight temperature dependence which might indicated a competition between temperature dependent and temperature independent length scales.

To conclude we have shown that the width $\Delta\nu$ and the derivative $(d\sigma_{xy}/d\nu)^{\max}$ in the high Landau levels of graphene show a typical scaling behavior with $\kappa=0.40\pm0.02$ leading to a critical exponent $\gamma=2.5\pm0.2$. These results are also confirmed by scaling measurements in the variable range hopping regime yielding $\gamma=2.6\pm0.5$. In the zeroth Landau level no temperature dependence of $\Delta\nu$ nor of $(d\sigma_{xy}/d\nu)^{\max}$ was observed in the measured range. This absence of a universal scaling behavior points toward a significantly different localization mechanism governed by a temperature independent intrinsic length scale.

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*u.zeitler@science.ru.nl

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